Galilean Islands in Eternally Inflating Background

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Abstract

We show that the observational universe may emerge from a de Sitter background with low energy scale, without requiring a large upward tunneling. We find, after calculating the curvature perturbations, that the corresponding scenario is actually a style of the eternal inflation, in which some regions will go through the slowly expanding Galilean genesis phase with rapidly increasing energy density and become island universes, while other regions are still in the eternal inflation.

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During the eternal inflation [1],[2],[3],[4], an infinite number of universes will be spawned. It is generally thought that inside an observational universe, a phase of the slow roll inflation and reheating is required, which set the initial conditions of the big bang evolution, i.e.the homogenizing and the scale invariant primordial perturbation.

In principle, the slow roll inflation should occur in a high energy scale, which is required to insure that the amplitude of primordial density perturbation is consistent with the observations and as well as after the slow roll inflation, the reheating temperature could be suitable with a hot big bang evolution. In this sense, it seems that the energy scale of the eternal inflation should be enough high, or the spawning of observational universe will be islandlike, which requires a large uptunneling [5],[6],[7], and is exponentially unfavored, e.g.[8].

However, the observational universe might classically emerge from a background with low scale, e.g. the emergent universe scenario [9], in which the universe originates from a static state in the infinite past. When the universe emerges, or begins to deviate from the Minkowski space, it is slowly expanding. In Ref.[10], it was for the first time observed that the slow expansion might adiabatically generate the scale invariant curvature perturbation, also [11],[12],[13]. The initial conditions of the big bang evolution may be set in this slowly expanding phase. During the slow expansion [10],[12], the null energy condition is violated.

Recently, the applications of Galileon [14], or its nontrivial generalization, e.g. [15],[16],[17], to the early universe have acquired increasing attentions, e.g. [18] for bouncing universe, and [19],[20] for Galilean genesis, in which the violation of null energy condition can be implemented stably, there is not the ghost instability. In Refs. [20], it was showed by applying the generalized Galileon that the scale invariant curvature perturbation may be adiabatically generated in slowly expanding Galilean genesis phase.

Here, inspired by [9], we would like to ask a significant question, whether and how the observational universe may classically emerge from a de Sitter background with low scale, and what about its scenario.

We will show it with a detailed model. We find, with the calculations of the perturbations, that the corresponding scenario is actually a style of the eternal inflation, in which some regions will go through the Galilean genesis phase and become island universes, while other regions are still in the eternal inflation. We argue that this scenario may be a viable design of the early universe, which might help to improve the current understanding to some things

in the eternal inflation.

We begin with the generalized Galileon as follows

$$\mathcal{L} \sim -e^{4\varphi/\mathcal{M}} X + \frac{1}{\mathcal{M}^8} X^3 - \frac{1}{\mathcal{M}^7} X^2 \Box \varphi - \Lambda. \tag{1}$$

where $X = \partial_\mu \varphi \partial^\mu \varphi/2$, Λ is the energy scale of the de Sitter background and $\mathcal{M}$ is constant. There are not the instabilities in perturbation level, as will be showed, though φ is ghostlike.

The evolution of background is determined by [20]

$$\begin{aligned} & \left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8} X^2 + \frac{24}{\mathcal{M}^7} H \dot{\varphi} X\right) \ddot{\varphi} \\ + & \ 3\left(-e^{4\varphi/\mathcal{M}} + \frac{3}{\mathcal{M}^8} X^2\right) H \dot{\varphi} \\ + & \ \left(-\frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} + \frac{6\dot{H}\dot{\varphi}^2}{\mathcal{M}^7} + \frac{18 H^2 \dot{\varphi}^2}{\mathcal{M}^7}\right) X = 0, \end{aligned} \tag{2}$$

$$3H^2 M_P^2 = -e^{4\varphi/\mathcal{M}} X + \frac{5}{\mathcal{M}^8} X^3 + \frac{6}{\mathcal{M}^7} X \dot{\varphi}^3 H + \Lambda. \tag{3}$$

When $H > \frac{\dot{\varphi}}{\mathcal{M}}$, the universe is in eternally inflating regime, and the background is highly inhomogeneous, as will be confirmed. Here, we require initially

$$H \ll \frac{\dot{\varphi}}{\mathcal{M}}. \tag{4}$$

Thus Eq.(2) is approximately

$$\left(-e^{4\varphi/\mathcal{M}} + \frac{15}{\mathcal{M}^8} X^2\right) \ddot{\varphi} - \frac{4}{\mathcal{M}} e^{4\varphi/\mathcal{M}} X \simeq 0. \tag{5}$$

The solution is

$$e^{\varphi/\mathcal{M}} = \left(\frac{5}{4}\right)^{1/4} \frac{1}{\mathcal{M}(t_* - t)}. \tag{6}$$

$$\dot{\varphi} = \frac{\mathcal{M}}{(t_* - t)}. \tag{7}$$

This implies $e^{4\varphi/\mathcal{M}} X = \frac{5}{\mathcal{M}^8} X^3$. Thus Eq.(3) is simplified as,

$$H^2 M_P^2 = \frac{H M_P^2}{x^4 (t_* - t)} + \Lambda/3, \tag{8}$$

where $x = \sqrt{\mathcal{M} M_P}(t_* - t)$ is defined, initially $|t| \gg |t_*|$ implies $x \gg 1$.

We define $H_0 = \sqrt{\frac{\Lambda}{3}}/M_P = -1/t_0$, obviously $|t_0| \gg |t_*|$. We define the critical time t_C as the time of $\frac{H M_P^2}{x^4 (t_* - t)} = \Lambda/3$ in Eq.(3), which is

$$t_C = \left(\frac{H_0^4}{\mathcal{M}^2 M_P^2}\right)^{1/5} t_0. \tag{9}$$

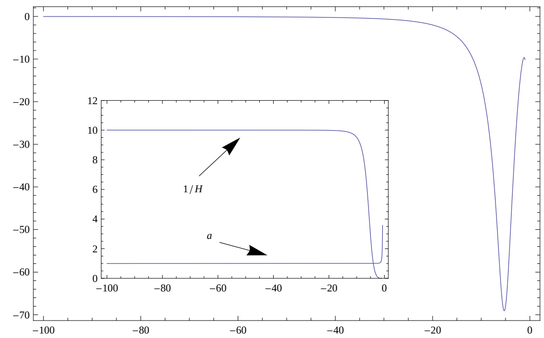

FIG. 1: The evolution of ϵ with the parameters $\mathcal{M} = M_P$ and $\Lambda = M_P^4/10^8$. The unit of horizonal axis is $1/\sqrt{\mathcal{M}M_P}$. Here, $t_C \simeq -6$ is given by Eq.(9), and $t_{Eter} = -10^2$ given by Eq.(29). In the regime $t < t_{Eter}$, the universe is in eternally inflating phase. Thus the time axis begins around -10^2. When $t_{Eter} < t < t_C$, the universe slowly expands with constant $H = H_0$, and when $t_C < t < t_*$, the universe is in slowly expanding Galilean genesis phase with rapidly increased H. The inset panel is the evolutions of a and $1/H$ with same parameters, in which initially $a = 1/H_0$ but for clarity a has been divided by 10^4 and $1/H$ by 10^3.

The corresponding value of φ is obtained, with Eq.(6),

$$\varphi_C \sim -\mathcal{M} \ln \left(\frac{\mathcal{M}^3}{M_P^2 H_0} \right)^{1/5} . \tag{10}$$

When $t < t_C$, $H = H_0$. Thus $a \sim e^{H_0 \Delta t}$, which seems indicate that the universe is exponentially expanding. However, since $H_0 \Delta t < 1$, i.e. the time that this phase lasts is shorter than one efold, a is almost unchanged. Thus in this regime the universe is actually slow expanding.

In certain sense, this phase and the slow expansion studied in Refs.[11],[13] is alike, also the slow contraction [21]. The difference is the tilt of the perturbation spectrum, as will be noticed.

When $t > t_C$,

$$H \simeq \frac{1}{x^4 (t_* - t)} \tag{11}$$

is rapidly increasing. Thus

$$a \sim e^{\int H dt} \sim Exp\left(\frac{1}{x^4} \right). \tag{12}$$

In this regime, $x \gg 1$, thus the universe is still slow expanding.

The definition of ϵ is $\frac{d}{dt} 1/H$, in which H is determined by Eq.(3), which is slightly

complicated. However, for different phases, we approximately have

$$|\epsilon| \simeq \frac{\mathcal{M}M_P}{H_0^2}/x^6, \quad in \text{ the phase } t < t_C, \tag{13}$$

$$x^4, \quad in \text{ the phase } t > t_C. \tag{14}$$

Initially the universe is in a dS phase with $H = H_0$ and $\dot{H}$ being gradually increased, thus initially $|\epsilon| \sim 0$ and will become larger and larger, while $t > t_C$ the universe is in the slowly expanding Galilean genesis phase with rapidly increasing H, in this phase $|\epsilon|$ will smaller and smaller.

We plot the evolution of background in Fig.1, in which the parameters $\mathcal{M} = M_P$ and $\Lambda \sim M_P^4/10^8$, We, with Eq.(9), have $t_C \sim -6$, which is consistent with Fig.1. Here, the parameters used are only to conveniently plotting the background evolution. In principle, we could have a broader choice of the range of parameters, e.g. Λ equal to or smaller than the value of the current cosmological constant.

We will calculate the curvature perturbation $\mathcal{R}$. The quadratic action of $\mathcal{R}$ is

$$S_2 \sim \int d\eta d^3x \frac{a^2 Q_{\mathcal{R}}}{c_s^2}\left(\mathcal{R}'^2 - c_s^2(\partial\mathcal{R})^2\right). \tag{15}$$

This has been calculated in the uniform field gauge, e.g. Refs.[16],[17]. Here, we have [20]

$$Q_{\mathcal{R}} = \left(\frac{-e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} + \frac{8X}{\mathcal{M}^7}(\ddot{\varphi} + H\dot{\varphi}) - \frac{8X^4}{\mathcal{M}^{14}M_P^2}}{\left(\frac{2\dot{\varphi}X^2}{\mathcal{M}^7} - H\right)^2}\right) X, \tag{16}$$

$$c_s^2 = \frac{-e^{4\varphi/\mathcal{M}} + \frac{3X^2}{\mathcal{M}^8} + \frac{8X}{\mathcal{M}^7}(\ddot{\varphi} + H\dot{\varphi}) - \frac{8X^4}{\mathcal{M}^{14}M_P^2}}{-e^{4\varphi/\mathcal{M}} + \frac{15X^2}{\mathcal{M}^8} + \frac{12H\dot{\varphi}^3}{\mathcal{M}^7} + \frac{12X^4}{\mathcal{M}^{14}M_P^2}}. \tag{17}$$

$Q_{\mathcal{R}} > 0$ and $c_s^2 > 0$ are required for the avoidance of the ghost and gradient instabilities.

The equation of $\mathcal{R}$ is

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) u_k = 0, \tag{18}$$

after $u_k \equiv z\mathcal{R}_k$ is defined, where $'$ is the derivative for $\eta = \int dt/a = t/a$, $z^2 \equiv 2a^2 Q_{\mathcal{R}}/c_s^2$.

When $k^2 \simeq z''/z$, the perturbation mode is leaving the horizon, and hereafter it freezes out. We call this horizon as the $\mathcal{R}$ horizon, which is approximately

$$1/H_{\mathcal{R}} = a\sqrt{\left|\frac{z}{z''}\right|} \sim t_* - t \tag{19}$$

for both phases $t < t_C$ and $t > t_C$, since a is approximately unchanged in both phases.

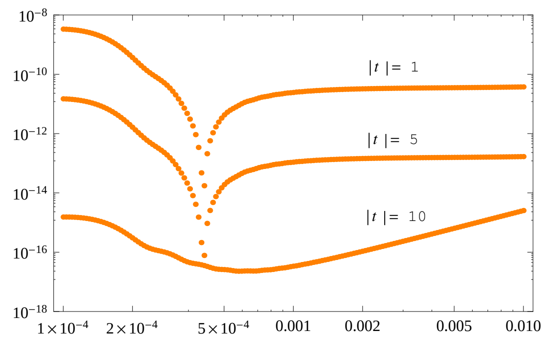

FIG. 2: The evolution of the perturbation spectrum with the time is plotted. The horizonal axis is k. The perturbation spectrum outside of the horizon is almost scale invariant for $k > k_C$, and is $\sim 1/k^4$ for $k < k_C$, the amplitude of perturbation spectrum increase with the time, but the shape of the spectrum is not altered. However, at $|t| = 10$ the spectrum is tilt for $k > k_C$, since the corresponding perturbation modes have not left the horizon and the spectrum is set by the initial condition.

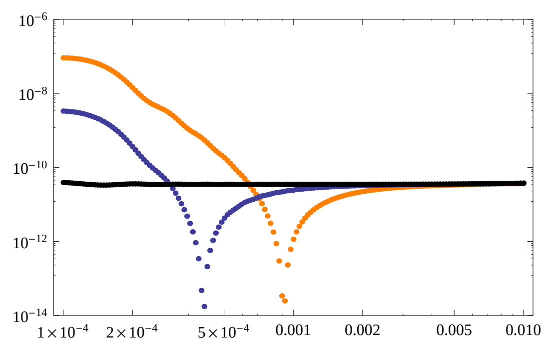

FIG. 3: The perturbation spectrum is plotted for the Langrangian (1) with $\Lambda = 0$ (black line), $\Lambda = M_P^4/10^8$ (blue line) and $\Lambda = 5 \times M_P^4/10^8$ (orange line), respectively. The horizonal axis is k. The spectrum is scale invariant for $\Lambda = 0$, but when $\Lambda \neq 0$ the perturbation spectrum on scale larger than $1/k_C$ will become highly red tilt $\sim 1/k^4$, and the larger Λ is, the larger k_C is.

Before we numerically solve Eq.(18), it is significant to see the approximate solution of Eq.(18). We approximately have

$$Q_{\mathcal{R}} \simeq \frac{\mathcal{M} M_P^3}{H_0^2}/x^6, \;\; in \text{ the phase } \; t < t_C, \tag{20}$$

$$M_P^2 x^4, \;\; in \text{ the phase } \; t > t_C, \tag{21}$$

and $c_s^2 \sim 1.4$. Thus although the Galileon (1) seems ghostlike, it is actually ghostfree for

the background evolution (3). Here, $Q_{\mathcal{R}} \simeq M_P^2|\epsilon|$ is noticed.

When $k^2 \gg z''/z$, i.e. the perturbation is deep inside the $\mathcal{R}$ horizon, u_k oscillates with a constant amplitude. The quantization of u_k is well defined for $Q_{\mathcal{R}} > 0$, which sets its initial value, $u_k \sim \frac{1}{\sqrt{2k}} e^{-ik\eta}$.

Eq.(18) is approximately a Bessel equation for $k < k_C$ and $k > k_C$, respectively. Thus when $k^2 \ll z''/z$, the solution of u_k is

$$u_k \simeq \frac{4e^{i3\pi/2}\Gamma(\frac{7}{2})}{\sqrt{2k}\Gamma(\frac{3}{2})} / \left(k(\eta_* - \eta)\right)^3 , \quad for \;\; k < k_C, \tag{22}$$

$$\frac{e^{i\pi/2}}{\sqrt{2k}} / \left(k(\eta_* - \eta)\right) , \quad for \;\; k > k_C. \tag{23}$$

The perturbation spectrum is $\mathcal{P}_{\mathcal{R}} \simeq k^3 \left|\frac{u_k}{z}\right|^2$, which is

$$\mathcal{P}_{\mathcal{R}} = \frac{\mathcal{M}^2}{H_0^2}\left(\frac{k_0}{k}\right)^4 , \quad for \;\; k < k_C, \tag{24}$$

$$\frac{\mathcal{M}}{M_P x^6}, \quad for \;\; k > k_C \tag{25}$$

where $k_0 = aH_0$ is the wavenumber of the perturbation mode leaving the horizon at certain time t_0. We see that in the region $k < k_C$, the spectrum is highly red tilt, while in the region $k > k_C$, it is scale invariant, but its amplitude will increase $\sim 1/x^6$. We also have $\mathcal{P}_{\mathcal{R}}(k > k_C) \simeq |\epsilon|\frac{H^2}{M_P^2}$, in which Eqs.(11) and (14) are applied.

The evolution of $\mathcal{R}$ outside the $\mathcal{R}$ horizon is

$$\mathcal{R} \sim D_1 \quad is \;\; \text{constant mode} \tag{26}$$

$$or \;\; D_2 \int \frac{d\eta}{z^2} \quad is \;\; \text{changed mode,} \tag{27}$$

where D_2 mode is increasing or decaying dependent of the evolution of z. In the phase $t < t_C$ the spectrum of $\mathcal{R}$ is dominated by the constant mode. While in the phase $t > t_C$ the spectrum of $\mathcal{R}$ is dominated by the increasing mode, we have $\mathcal{R} \sim \int d\eta/Q_{\mathcal{R}} \sim 1/x^3$, which is consistent with (25). In this phase, for the perturbation mode outside the horizon, only is its amplitudes increasing, but the shape of the spectrum is not altered [23],[24].

Here, a distinct thing is though in the phase $t < t_C$ the amplitude of perturbation having left the horizon is constant, it will synchronously increase with the perturbation leaving the horizon in the phase $t > t_C$ after $t > t_C$.

When $|\epsilon| \sim 1$ or $x \simeq 1$, the change of a begins to become not negligible. Though $|\epsilon|$ is still decreasing, a is increased exponentially. Thus the increasing of the perturbation amplitude

will come to a halt at certain time $t_e \sim \mathcal{O}(t_*)$ shortly after $|\epsilon| \sim 1$. Thus the spectrum of $\mathcal{R}$ should be calculated around t_e,

$$\mathcal{P}_{\mathcal{R}} \sim \frac{\mathcal{M}}{M_P}, \tag{28}$$

where $x_e \sim 1$ is applied. $\mathcal{P}_{\mathcal{R}}^{1/2} \sim 1/10^5$ requires $\mathcal{M}/M_P \sim 1/10^{10}$.

We numerically solved Eq.(18) with the numerical solutions of Eqs.(2) and (3), and plotted the evolution of the amplitude of the curvature perturbation in Fig.2 and the resulting perturbation spectra for the different values of Λ in Fig.3.

We see that in Fig.2 the perturbation spectrum is almost scale invariant for $k > k_C$, and is $\sim 1/k^4$ for $k < k_C$, the amplitude of perturbation spectrum increases with the time, but the shape of the spectrum is not altered. Figs.2 and 3 are consistent with analytical results (24) and (25).

In the region $k < k_C$, $\mathcal{P}_{\mathcal{R}}$ is highly red tilt, which implies that its amplitude will rapidly increases with scale $1/k$. When $k = \sqrt{\mathcal{M}H_0}$, we find $\mathcal{P}_{\mathcal{R}} \sim 1$. The corresponding time and the field value are, respectively,

$$t_{Eter} \sim -\frac{1}{\sqrt{\mathcal{M}H_0}} \sim \sqrt{\frac{H_0}{\mathcal{M}}}t_0, \tag{29}$$

$$\varphi_{Eter} = \mathcal{M}\ln\frac{1}{\mathcal{M}(t_* - t)} \sim -\mathcal{M}\ln\left(\frac{\mathcal{M}}{H_0}\right). \tag{30}$$

Thus in the phase $t < t_{Eter} = \sqrt{\frac{H_0}{\mathcal{M}}}t_0$ or $\varphi < \varphi_{Eter}$, the energy density ρ_φ of local regions will be in a randomly walking state. This in certain sense implies that the global universe is in an eternal inflating regime, i.e. some regions have gone or are going through the Galilean genesis phase, but other regions are still in inflationary phase, the inflation never completely ends.

We will calculate the fluctuation of the Galileon field φ, following Ref.[22]. When $k^2 \ll z''/z$, $\delta\varphi_k$ is

$$\delta\varphi_k \simeq \frac{\Gamma(\frac{5}{2})e^{i\pi}}{a\sqrt{2kQ_{\delta\varphi}}\Gamma(\frac{3}{2})} / \left(k(\eta_* - \eta)\right)^2, \tag{31}$$

where $Q_{\delta\varphi} \simeq \frac{M_P^2}{\mathcal{M}^2}/x^4$. Thus the average square of the amplitude of field fluctuations is

$$< \delta\varphi_k^2 >= \frac{1}{8\pi^3}\int_{H_{\mathcal{R}}/e}^{H_{\mathcal{R}}} |\delta\varphi_k|^2 \, d^3k \sim \frac{\mathcal{M}^4}{H_{\mathcal{R}}^2}, \tag{32}$$

where $k \sim H_{\mathcal{R}} = 1/(t_* - t)$ is applied for the perturbation being leaving the horizon. The classical rolling of Galileon field is approximately

$$\Delta\varphi = \dot{\varphi}/H_0 \sim \frac{\mathcal{M}H_{\mathcal{R}}}{H_0}, \tag{33}$$

where $\dot{\varphi}$ is given by Eq.(7). Thus for $k < \sqrt{\mathcal{M}H_0}$, we have

$$\frac{\sqrt{< \delta\varphi_k^2 >}}{\Delta\varphi} \sim \frac{\mathcal{M}H_0}{H_{\mathcal{R}}^2} > 1. \tag{34}$$

Thus when $t < t_{Eter}$, i.e. $\varphi < \varphi_{Eter}$, the perturbation of field is larger than its classical rolling, which implies that the field is randomly jumping. Thus in some local regions of the global universe, the field will jumped to the regime of $\varphi > \varphi_{Eter}$, and begin the evolution of slowly expanding Galilean genesis, while in other regions the field will again jump back and is still in random jumping. This result again indicates that the global universe is in an eternally inflating regime.

When $t \sim t_{Eter}$, we have

$$\dot{\varphi}/\mathcal{M} = \sqrt{\frac{\mathcal{M}}{H_0}}H_0 \gg H_0, \tag{35}$$

which insures the rationality of (4).

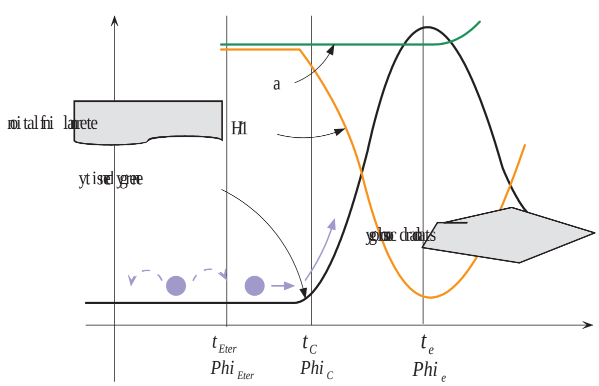

FIG. 4: The Galilean island in an eternally inflating background.

We summarized this scenario in Fig.4. During the eternal inflation, some regions of global universe will go through the slowly expanding Galilean genesis phase and become island universes, while other regions are still in eternal inflation, which will make the room for more island universes to emerge. The island universes generally has the initial conditions $a_{ini} \simeq 1/H_0$ and $H_{ini} = H_0$, which is consistent with Ref.[25]. When $t_{Eter} < t < t_C$, i.e. $\varphi_{Eter} < \varphi < \varphi_C$, the local universe is in a slowly expanding phase with almost constant

$H = H_0$, the spectrum of the primordial perturbation generated in this phase is highly red tilt. While when $t_C < t < t_e$, the universe is in the Galilean genesis phase with rapidly increasing H, the spectrum may be scale invariant. After the genesis phase of local universe ends, the available energy of field is released to reheat the universe, e.g.[26],[20]. Hereafter, in the corresponding local region, or islands, the evolution of hot big bang model begins.

Here, we only show one among the implements of this scenario. The implement with the conformal Galileon, as well as DBI genesis [27], is essentially similar, though the adiabatic perturbation is not scale invariant, the obtaining of the scale invariant curvature perturbation may appeal to either the conversion of the perturbations of other light scalar fields, e.g. conformal mechanism [28],[29], or a period of inflation after the emergence of Galilean islands. The relevant issues are also interesting for investigating.

Here, though Λ is constant, the result actually has captured the physics of this scenario. However, the model with Λ being a landscape of effective potential of Galileon field φ or other fields is certainly far interesting. This issue will be studied in upcoming work.

In the discussions of the measure problem for eternally inflating multiverse, e.g. [30],[31], it is generally thought that the emergence of islandlike universe requires a large uptunneling, which is exponentially unfavored. In certain sense, the scenario showed here is a significant supplement to the phenomenology of the eternal inflation, which might help to improve the understanding for the measure problem. The relevant issue will be discussed elsewhere.

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